Design & Pricing of Islamic Asset-Based Ṣukūk Strips: An Endeavour towards Financial Development

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Keywords
Ṣukūks
Financial Intermediation
Liquidity Management
Financial Innovation
Financial Development

Abstract.
Purpose: The efficiency of the Islamic Financial Industry suffers from a lack of liquidity instruments and shallow secondary markets. This paper reviews the financial intermediation theory and the complementary role of debt and money markets towards the broader goal of allocative efficiency, asset transformation, and diversification for the welfare of all economic agents. Separately Tradable Registered Interest & Principle (STRIPs), created by disintegrating coupon and principal payment to be traded as discount bonds, have been effectively employed in the west for liquidity management. This paper proposes a novel idea of ijārah ṣukūk STRIPS (ISS) that splits the payoffs of ijārah ṣukūk.

Method: The design and pricing of ISS are modeled employing lease valuation, certainty equivalent (C.E.), and bootstrapping. Instead of the prevalent capital asset-pricing model, volatility of ijārah ṣukūk payoffs is modeled using certainty equivalents, which extend the analysis to micro-level investor utility function. Also, the viability of ISS design is substantiated by numerical illustrations.

Findings: We found that the proposed ISS can serve as a short-term liquidity management instrument, facilitate price discovery and define the term structure in Islamic debt markets. For sovereign issues, ISS can enable efficient monetary policy transmission, market completeness, and public finances for the underdeveloped Muslim world.

Significance: To the best of our knowledge, this is the first study proposing the ISS as a financial innovation. The ISS proposed in this study will be a valuable addition to the arsenal of Islamic Financial offerings - augmenting the financial deepening and development of Islamic Financial Markets.

KAUJIE Classification: H13, K1, K10, L32, L41, L42
JEL Classification: G23, O16

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INTRODUCTION

The four-decade-old Islamic Finance Industry has exhibited a prolific growth starting from a modest initiative of Mit Ghamr in Egypt in the early sixty’s and Dubai Islamic Bank back in the mid-seventies, to the present day, where 430 Islamic Financial Institutions (IFIs) and 191 Islamic Window operation are in operation across 78 countries, with assets worth over USD 1.8 Trillion under its folds. Despite this exuberant proliferation, the industry remained bank-centric, with 80.3% of its total assets on the balance sheets of Islamic Banks (Kammer et al., 2015).

Secondary markets remained thin and illiquid, which is unequivocally attributable to the lack of market depth and incompleteness of product offerings (Iqbal, 2007). The lack of deep debt and money markets and scarcity of tradable liquidity management instruments incited short-termism in Islamic Banks, with most of its funding parked in short-term trade-based financing (Alamsyah & Masih, 2017; Meisame et al., 2017). This trend obstructed the core purpose of asset transformation aspired by a financial intermediation system (Abu-mounes, 2020). Review of financial intermediation theory underscores the mutually reinforcing role of banks and capital markets to achieve temporal and spatial asset transformation and subsequent allocative efficiency of economic resources (Ahmed, 2016). The existing Islamic money market product suite, primarily based on tawarrug, commodity murābahah, and bai‘-al-‘inah is deemed controversial and inefficient due to its non-tradability and irrational adherence to trade-based contracts (Saleem et al., 2021). Though the recent rise in the long-term sovereign ijārah based šukūk issues has pacified the situation to some extent; however, liquidity in the secondary markets remains thin due to a lack of flexibility in term structures, supply constraints, and inadequate market-making arrangements (Selim et al., 2019).

Economic historians attribute the industrial revolution vis-a-vis the economic development of the west to the financial development - which enabled the efficient flow of capital to fund long-term and illiquid production technology while simultaneously managing to match the liquidity preferences of their investors (Ahmed, 2016). This underscores the exigency of financial engineering to promote financial deepening in the so-called Islamic Financial system (Beck et al., 2013). The way forward lies in transitioning from a primitive bank-centric form of intermediation towards an efficient direct and indirect form of financial intermediation (Ebrahim & Hussain, 2010).

Given the above, this paper aims to pursue two key objectives, i.e. (i) to evaluate the role of money markets instruments and liquidity management solutions practiced in the Islamic Financial Markets and the underlying Shari‘ah compliance challenges; (ii) to model the design, structure, and pricing, and illustrate the Shari‘ah rationalization of ISS instruments.

The study is divided into four sections. A literature review is presented in section 2 - expounding on the role of financial deepening and specifically money markets towards an efficient financial intermediation system. Following a logical progression, the literature review assesses the mainstream liquidity management products, including money market instruments to compare and contrast them with practices in Islamic Financial System. The focus is then narrowed down to the business case of innovation of ījārah šukūk STRIP.
instrument. Section 3 firstly sets the underlying theoretical considerations influencing the design of an Islamic Money market and subsequently articulates the mathematical model to value the proposed *ijārah* based zero-coupon bonds. The fourth and the last section concludes the study.

**LITERATURE REVIEW**

**The Business Case of the Financial Innovation - Treasury STRIP**
The US-Treasury STRIP, a three-decade-old innovation, offers a clear case of financial innovation, satisfying investors’ demand. Back in the early 1980s, an investment banker at Merrill Lynch and Salomon Brothers synthesized a financial security which was effectively an equivalent of zero-coupon bonds (Zeros) by "stripping" Treasury issues-i.e., by separately selling the coupons from the principal payments. It was later in 1985, adapted by U.S. Treasury itself, and was branded as Separate Trading of Registered Interest and Principles (STRIPS). This novelty triggered the underlying riskless arbitrage opportunity in case of inconsistencies in the notes, bonds, and STRIPS prices. For example, if the sum of the present values of the coupons and principle Zeros exceeds the bond’s market price, there is an incentive to strip.

The striping of the varying maturity of the treasury issues facilitates market completeness, as discount bonds for a specific future maturity can only be synthesized by striping (Ahmad & Lal, 2006; Grinblatt & Longstaff, 2000). Although both corporate and Treasury debt issues are strippable, but many investors have a preference for treasury Bonds given their liquidity, zero risk, marketability, and non-callability. It is worth noting that empirical studies have found STRIP Zeros to enhance liquidity, besides being resilient in the recent financial crisis (Vonhoff 2014).

**Islamic Money Market Instruments and Liquidity Management Practices**
The necessity to foster short-term liquidity management instruments and money markets is of utmost significance for the progress of bank-centric Muslim economies. Currently, Islamic financial markets predominantly employ organized *tawarruq*, commodity *murābaḥah*, inter-bank *wakālah*/*wadī`ah-hiba*/*muḍārahah* arrangements to manage their liquidity management objectives. Relatively in developed markets, the central banks have initiated programmed issuances of instruments based on short-term *muḍārahah*, *salam*, *istiṣnā‘*, *murābaḥah*, *bai‘-al-‘īnah* to absorb excess liquidity in the banking sectors (Kahf & Hamadi, 2014). Table-1 below presents the Islamic liquidity management instruments classified based on Sharī‘ah contracts and the practising jurisdiction.

The modes discussed above are limited by their non-tradability (based on trade-based receivables assets) (AAOIFI, 2010). They have also been criticized for being based on ḥiyals (artifact) and hence suffer from controversy and acceptability (IRTI, 2014). This prohibition is specially well-pronounced in the case of Reverse-*murābaḥah*, Organized *tawarruq* (OIC Fiqh Academy, Resolution 179/19), and *bai‘-al-‘īnah* based liquidity management instruments (OIC Fiqh Academy, Resolution 157/17), employed by central banks in Bahrain, Kuwait, and Malaysia (Bacha et al., 2013).
**Ijārah Based Securitization and Liquidity Management**

The natural tilt of the issuer and the investor is towards *ijārah* based *ṣukūk*, owing to its ex-ante return determinability and better investor protection and recourse in terms of asset ownership and repossessibility. Moreover, the tradability of *ijārah* *ṣukūk* and the least Sharīʿah discontent reverberated in it, add its favourability. Nevertheless, the practice of *ijārah* structures has also been subjected to censure by the Sharīʿah cohorts when a sensational ruling surfaced by AAOIFI in 2008. The ruling primarily insisted on the true sale, ensuring legal ownership and the risk of the asset to be transferred to the SPV instead of an exchange of mere monetary flows and financial claims. This obligation of true sale demarcates the difference between ‘asset-backed’ and asset-based instruments, wherein the former is deemed tradable in the view of the Sharīʿah (Bank Negara Malaysia, 2010; IFSB, 2008; IIFM, 2011).

**Rationalizing Sharīʿah Acceptability of Ijārah Ṣukūk STRIPs**

The preliminary injunctions of Islamic finance frown upon economic arrangements, entrenching *ribā* (translated as usury but with a literal meaning of ’excess’), *gharar* (uncertainty of outcomes), and *maysir* (gambling) or operating in business involving *ḥarām* (prohibited) goods or services. Avoidance of *ribā* in form and substance warrants a rigorous understanding of its broader implications and the underlying wisdom (Haider et al., 2021). Ebrahim et al. (2014) professes *ribā* emanating from a plain vanilla debt instrument lies in the endemic agency issue, leading to expropriation of wealth/property rights, fragility in economy, and financial exclusion of the poor. The Sharīʿah underpinnings warrant risk-sharing arrangements for the investor to earn the profits (Iqbal et al., 1998). El-Hawary et al. (2004) succinctly articulated the basic principle to qualify a Sharīʿah-based product, encompassing risk sharing, linkage to the real economy, non-expropriation, and avoidance of sinful financing.

The *ijārah* *ṣukūk* STRIP (ISS) instrument presented in this study is founded on lease transaction, which fulfills the fundamental doctrine of Sharīʿah and curbs agency cost of debt (Ebrahim et al., 2015). Borrowing the hermeneutic approach of *usuli* rationalization based on the overarching *maqāṣid* (aims) of Islamic Law, the *ijārah* STRIP instruments methodology and its ramification can be validated on the following principles of (i)’Securing welfare *jalbal – masāliḥ* and deterring damage (*daff al – ḍharar*); (ii) Bringing ease (*taysīr*) and limiting adversities (*rafi al – ḥaraf*); (iii) Freedom to contract (*ḥurūfiyat al – taʾāqud*) (iv) Cooperation (*taʾāwun*) and Conflict Aversion (*daff al – ‘adāwah*). As deliberated in the preceding sections, the *ijārah* STRIP instrument is a contractual arrangement expected to augment Muslim countries’ financial and economic development, and facilitate liquidity management and allocative efficiency across IFIs and corporates. Accordingly, in the wake of the above rationalization, the legal permissibility (*mubāḥ*) of ISS may be validated.
DESIGN, STRUCTURE, AND MODELLING IJĀRAH ṢUKŪK STRIP SECURITIES

Methodology
To present the design and structuring of an innovative product based on financial engineering, this paper opts for a theoretically motivated mathematical modelling (backed by numerical illustrations) approach to fulfil its research objective.

Key Concepts, Theoretical Underpinnings & Design Consideration
In the conventional economic arrangement, the Treasury securities issued and backed by the Government set the base rate which is the minimum rate of return warranted by the investors. This base rate serves as a pricing benchmark for private debt issues, which are expected to yield a premium over and above the base rate. The premium or the spread riskiness is grounded in the type and creditworthiness of the issuer, term to maturity, tax treatment, and liquidity of the issue itself. Markets developed a graphical depiction of the yield vis-a-vis maturity of the treasury securities and termed it a ‘Yield Curve.’ The discerning yield of a coupon bond versus a discount bond of the same maturity brings in the idea of spot rate and its graphical rate, which is termed the ‘Term Structure of Interest rate’ or ‘Term Structure of Ṣukūk Profit Rates’ in IMM.

The Implication of Term structure of Profit Rates on Ṣukūk Striping
The incentive to strip or reconstitute is essentially contingent on the term structure of profit rates. As evident in the figure below, a rising term structure offers an advantage to strip a Ṣukūk, while reconstitution is favorable in case of a declining term structure. Moreover, the profit from striping a Ṣukūk increases with increasing maturities in an increasing term structure. The figure below summarizes the positive impact of rising term structure and maturity over the price of STRIP relative to their underlying Ṣukūk.

Mathematically represented, if $P_s(y,t)$, $P_l(y,t)$ and $P_p(y,t)$ denote the price of a Ṣukūk, its
corresponding rental and terminal payoff STRIP respectively for a given yield and maturity, then \( a(y,t) \) represents the differential which triggers the arbitrage for striping a \( \text{sukūk} \) or reconstituting the STRIP portfolio.

It is pertinent to mention that \( a(y,t) \) increases with the increasing maturity and expected rate of return \( 'r.' \) In case of the rising term structure, i.e., when

\[
 r_t < r(t+1) < r(t+2) < r(t+n)
\]

Then, \( a(y,t) > 0 \); and vice versa for a declining term structure.

Whereas, for a flat term structure, i.e. when

\[
 r_t R(t+1) = r(t+n)
\]

Then, \( a(y,t) = 0 \), i.e., the case of no-arbitrage.

**Modelling Term Structure of Islamic Money Markets Instruments**

Understanding the principle of equilibrium asset pricing is of utmost importance to unleash the pragmatic adaptations of the theoretical foundations of the term structure of interest rates. In their attempt to model investor behavior, the theoretician insists that investors value money in terms of the utility of consumption. Two universally generalizable features characterized are the ‘Term premium’ and the decreasing marginal utility relative to increasing wealth levels of an individual investor (Van Der Merwe, 2015).

Thus the price of an asset is

\[
 P_t = E_t \left( \beta^{u(t+1)} x(t+1) \right)
\]

Or

Where \( \beta \) is the discount factor, less than one, and reflects the term premium, while \( x(t+1) \) is the wealth endowment. Hence, the ratio of marginal utility \( m_{t+1} \) of future time ‘\( t + 1 \)’ and the initial period that reflects the spot rate.

i.e. \( m_{t+1} = \left( \beta^{u(t+1)} \right) \)

The collection of these intertemporal yield rates of substitution, i.e., \( r_1, r_2, r_3, r_4, \ldots, r_n \), projects the spot rate curve and thus determines the term structure of interest rates.

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**FIGURE 2.** Yield Curve as a function of Intertemporal Rate of Substitution
Modelling Yield-to-Maturity, Spot, and Forward Rates

In the rest of the section, an attempt is made to mathematically represent and characterize the concepts of YTM, Spot, and Forward rates, built around the prior discussion on the term structure and its theoretical constructs.

Assuming a discount bond priced at ‘yt’ maturing at time ‘t,’ the value discount bond $B_z$, under risk-neutrality, can be modelled as:

$$B_z = E[M_t] = \frac{1}{(1+y_t)^t}$$

**Notation:**

- Let $i_{t(t1,t2)}$ be the Interest rate from time $t_1$ to $t_2$ prevailing at time $t$
- $P_{t0(t1,t2)}$ is the price of a bond quoted at $t = t_0$ to be purchased at $t = t_1$ maturing at $t = t_2$
- YTM is the yield to maturity which is the percentage increase in $s$, earned from the bond. Mathematically, YTM is the invert of the price of the zero-coupon bond.

$$i.e. 1 + YTM = \frac{1}{P_{t0(t1,t2)}} \Rightarrow YTM = [P_{t(t1,t2)}]^{-1} - 1$$

The concept of forward rates originates from the need to find the expected discount rates, at a future time period ‘t’ to a time period ‘t + 1’, i.e., for example, $r_{0(0,1)}$ and $r_{0(0,2)}$ are the expected rates of return for one year and two year zeros are known to us, then the expected forward rate $r_{0(1,2)}$ is the expected rate of return for the second year, one year from now. This is computed mathematically, assuming no-arbitrage. This can be mathematically represented below, enabling us to work out the forward rate $r'_{0(1,2)}$, as represented in the figure 8 below.

$$[1 + r_{0(0,1)}]E[1 + r_{0(1,2)}] = [1 + r_{0(0,1,2)}]2$$

Understanding the idea of the Term structure of sukūk profit rate is crucial for the process of striping, wherein the changing expectation of the investors in terms of growth and hence the intertemporal rate of substitution or the so-called the ’Spot rates.’

Carrying on the discussion towards modeling the dynamics of STRIP, as a process of disintegration of individual claims to the cash flows (i.e., to the lease rental ‘L’ and terminal value ‘P’), the valuation of a periodic rental paying ijārah sukūk can be computed, by aggregating the present value of its cashflows, is represented as (Williams, 1938):

$$B = \sum_{i=1}^{n} \frac{(L_i+P)}{(1+r_i)^i}$$

Whereas the YTM is the internal rate of return earned by an investor on a coupon bond, bought at the market price, and held till maturity (Bodie et al., 2007). Thus, valuing an ijārah based sukūk STRIP instrument requires us to primarily focus on the (I) Lease Payoffs (ii) required rate of returns.
Articulation of the *Ijārah Şukūk* STRIP Valuation Model

As defined earlier, the scope of this study is limited to the striping of *i jārah* / Leased based securities only. The valuation model encompasses the following key steps:

I. Modelling the term structure of default-free *şukūk* profit rates by employing Bootstrapping method (Fabozzi, 2007) to compute the required rate of return in section 3.8. A numerical example illustrates the exercise in section 3.8.1.

II. Valuation of *i jārah* Payoffs based on well-known Myers et al. (1976) model (BDM) in section 3.9.

III. Computation of Risk-adjusted cashflows by adapting the Certainty Equivalent Method (CEM) (Zhang, 2010) to value the risky payoffs of a leasing contract in sections 3.10 and 3.11. The valuation is further spelled out through Numerical illustrations in Section 3.11.1.

IV. Design and valuation of *i jārah şukūk* STRIP by discounting the risk-adjusted cash flow, i.e., the C.E. of lease payoff at the required rate of return, illustrated by a stylized diagram.

**Bootstrapping Methodology for Theoretical Spot Rate Curve**

The yield curve is the graphical depiction of the yield of the bonds of the same quality but different maturity. The primary focus of the study is the striping of *i jārah şukūks* with risky cashflows. A *şukūk* can be viewed as a bunch of distinct claims to future cashflows with varying maturities and thus need to be discounted at a unique rate of return called Spot rates. Therefore, in an arbitrage-free market, the sum of values of the zeros should ideally equate to the price of the *şukūk*. The arbitrage-free market is mathematically represented as:

Value of ‘n’ period Coupon Bond priced at market rates = Summation of its ‘n+1’ ZCBs priced at Spot yields

Or

\[ P_{(c,t)} = \sum_{i=1}^{n} \frac{L_i}{(1+y)^n} + \frac{P_0-D_{\text{econ}}}{(1+y)^n} = \sum_{i=1}^{n} \frac{L_i}{(1+Z_i)^n} + \frac{P_0-D_{\text{econ}}}{(1+Zn)^n} \]

Note: Here, the rates and Lease payment need to be doubled to arrive at the annualized yield values. Also the par value is assumed to be a hundred units for simplicity.

Since the observable Yield for Zeros is only limited to six months and one year’s maturity, the spot rate curve discussed above is created theoretically through a process called ‘Boot-
Strapping’ (Fabozzi, 2007). This methodology extrapolates the spot rate for a future period by incorporating the available spot rates (i.e., the 6-month and 1-year Zeros yield)

\[ P(c,3) = \frac{L_1}{[1+r_{(0,1)}]} + \frac{L_2}{[1+r_{(0,2)}]^2} + \frac{L_3}{[1+r_{(0,3)}]^3} \]

Where,

- \( P(c,t) \) is the price of the bond with Maturity ‘t’ (i.e., 1.5 years or three periods), and Lease periodic payoff is ‘Ln.’
- \( r_{0(0,1)} \) and \( r_{0(0,2)} \) represent the observable yield for six months and 1-year Zero Coupon Securities.
- \( Z_{0(0,3)} \) is the unknown variable of interest rate and represents the theoretical Spot rate to be used to price 18 Months Zero.

And finally, ‘\( P \)’ represents the expected par value that is assumed to be USD 100.

This bootstrapping process may be iteratively carried out to derive the spot rates across all maturity up to \( Z_{0(n)} \), given the spot rates, whether observed or computed, are available up to the previous ‘\( n-1 \)’ period. Thus, the generalized form of the Bootstrapping process can be mathematically expressed as:

\[ P(c,3) = \frac{C_1}{[1+r_{(0,1)}]} + \frac{C_2}{[1+r_{(0,2)}]^2} + \frac{C_3}{[1+Z_{(0,3)}]^3} + \ldots + \frac{C_{(n-1)}}{[1+Z_{(0,n-1)}]^{n-1}} + \frac{C_n + P_0 - D_{econ}}{[1+Z_{(0,n)}]^n} \]

**Numerical Illustration**

Continuing the four periods or two years stylized illustration in the previous sections, the bootstrapping process is illustrated, contemplating the respective unknown spot rate for third (1.5 years) and fourth period (2 years) zeros. The following hypothetical data is assumed for the Shari‘ah-compliant default-free Government Securities.

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Coupon rate</th>
<th>Yield-to-maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.000</td>
<td>0.080</td>
<td>96.15</td>
</tr>
<tr>
<td>1.00</td>
<td>0.000</td>
<td>0.083</td>
<td>92.19</td>
</tr>
<tr>
<td>1.50</td>
<td>0.085</td>
<td>0.089</td>
<td>99.45</td>
</tr>
<tr>
<td>2.00</td>
<td>0.090</td>
<td>0.092</td>
<td>99.64</td>
</tr>
</tbody>
</table>

Substituting the data in the equations above to derive the \( Z_{0(0,3)} \), i.e., the theoretical spot rate for 1.5 years ZCB.

\[ 99.45 = \frac{4.25}{[1+0.0400]} + \frac{4.25}{[1+0.0415]^2} + \frac{104.25}{[1+Z_{0(0,3)}]^3} \]
Therefore, in annualized percentage terms, 8.93% is the theoretical spot rate for 1.5 years ZCB.

Similarly, the Z(0,4) i.e. the theoretical spot rate for 2 years Zero can be worked out, by chipping in the previously found spot rates. A brief calculation is illustrated below:

\[ 99.64 = \frac{4.50}{1+0.0400} + \frac{4.50}{1+0.0415}^2 + \frac{104.50}{1+0.04465}^3 + \frac{104.50}{1+Z(0,4)}^4 \]

\[ \Rightarrow [1 + Z(0,4)]^4 = 1.198158 \]

\[ \Rightarrow Z(0,4) = 0.046235 \]

Thus, doubling the above figure, to arrive at the annualized yield, the spot rate for two years ZCB is found to be 9.247%.

This spot yields calculated above for 1.5 years, and two years Zero, is used to price the respective sukūk STRIPS based on the following relationship:

\[ P(0,n) = \frac{1}{1+Z(0,n)} \]

Therefore, the price of the respective PCBs are:

\[ P(0.3) = \frac{1}{1+0.04465} = 0.8772 \text{ per Dollar or } 87.72 \text{ for USD 100 expected face value.} \]

\[ P(0.4) = \frac{1}{1+0.046235} = 0.8346 \text{ per Dollar or } 83.46 \text{ for USD 100 expected face value.} \]

Thus, the hypothetical sukūk striping exercise and data presented above can be summarized in the table below, along with the pricing of ijārah sukūk STRIPS:

<table>
<thead>
<tr>
<th>Period-years</th>
<th>n</th>
<th>Default Free Securities Market Data</th>
<th>Spot Yield Z(0,n)</th>
<th>Forward Rate ZCB Price per Dollar</th>
<th>STRIPS Face Value</th>
<th>Price of STRIPs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 years</td>
<td>1</td>
<td>0.08</td>
<td>96.15</td>
<td>0.08</td>
<td>na</td>
<td>0.962</td>
</tr>
<tr>
<td>1 year</td>
<td>2</td>
<td>0.08</td>
<td>92.19</td>
<td>0.08</td>
<td>0.083</td>
<td>0.922</td>
</tr>
<tr>
<td>1.5 years</td>
<td>3</td>
<td>0.085</td>
<td>99.45</td>
<td>0.089</td>
<td>0.083</td>
<td>0.886</td>
</tr>
<tr>
<td>2 years-Rental</td>
<td>4</td>
<td>0.09</td>
<td>99.64</td>
<td>0.093</td>
<td>0.086</td>
<td>0.901</td>
</tr>
<tr>
<td>2 years-Principle</td>
<td>4</td>
<td>0.09</td>
<td>99.64</td>
<td>0.093</td>
<td>0.086</td>
<td>0.883</td>
</tr>
<tr>
<td>Summation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The previous stylized illustration of the striping of șukūk is reproduced here with the respective spot rate of return and the striping of a hypothetical ijārah șukūk into four discrete STRIP securities along with the respective prices, yield, and par values:

![Stylized Illustration of Ijārah Șukūk STRIPs](image)

**FIGURE 4.** Stylized Representation of the overall Design, Structure, and Valuation of Ijārah Șukūk STRIPs

The table above and the figure above summarize the idea and operationalization of șukūk STRIPs based on hypothetical data as discussed above. A two-year or four periods șukūk is dismantled into four Separately Tradable Money Market Instruments, with a defined yield (i.e., the Spot rates) and pricing. It is evident here that the Bootstrapping method employed here leaves no arbitrage opportunity, as the aggregation of the prices of STRIPs, i.e., 99.64 is equivalent to the cost of the underlying two years șukūk. However, the granularity thus created converts a long-term debt instrument into short-term money market instruments and a series of longer-term ZCBs. This subsequently leads to greater trading and liquidity and enables cashflow matching for portfolio managers, especially pension funds and fund managers.

**Valuing Ijārah Securities and Payoffs**

The well-known Myers et al. (1976) model (MDB) is employed to elaborate on the lease payoff. The MDB model, used for a single lease payment scenario, captures the post-tax returns by equating it with the sum of post-tax lease rentals, expected salvage value. These depreciation tax shields are netted-off for applicable operating expenses (insurance or maintenance) to be borne by the lessor, as represented in the equation below:

\[
[1 + E(r)(1-\tau)]Q = L(1-\tau) + D\tau + E(S\text{Salv}) - \sigma(1-\tau)
\]

As mentioned above, since the lessor or the ijārah investor bears the operating expenses, the net lease payment (net-of operating expense) can be represented as \( L'(1 - T) \)
Thus, the above equation may be rewritten as:

\[
Q[1 + E(\bar{r})(1 - \tau)] = L'(1 - \tau) + D_t + E(S_{t0})
\]

\[
\Rightarrow \bar{r} = E(\bar{r}) = \frac{[L'(1 - \tau) + D_t - Q]}{Q(1 - \tau)} + \frac{[E(S_{t0})]}{Q(1 - \tau)}
\]

The above equation can be iteratively solved for \( \bar{r} \) to calculate the ex-ante and ex-post yield by substituting the expected and realized values for the variables, respectively. Thus, the income from a ‘t’ period lease is stochastic, comprising of ‘t’ rental payoff \( L't \), which is mathematically evaluated (employing Myers et al., 1976, approach) as follows:

\[
L'_t = L_t(1 - T) + D_t T
\]

The terminal payoff of the lease payment would also have the expected liquidation/residual value of the leased asset, represented as ‘E(P0)’, which is adjusted for economic depreciation Decon:

\[
L' = L(1 - T) + D T + P_0 - D_{econ}
\]

Where \( T \) represents the tax rate and \( D_t T \), and \( L_t (1-T) \) is the depreciation write-offs and post-tax lease payoff for all periods, respectively.

**Rationalizing the choice of Certainty Equivalent Method (CEM)**

To model the price of the Zeros, it is imperative to recognize that the CAPM model operates on the rate of return, not prices. What if you wanted to price an asset with an uncertain (not predetermined) rate of return? The intuition of the CAPM is driven by the assumption of the asset being priced in a perfect market setting at their fair value, where the price is equal to the present value of the expected cash flows. The CAPM, in all its parsimony models the risk-adjusted required rate of return of an asset as a function of its Beta, relative to a market portfolio.

\[
\text{Expected return} = \text{Risk free Rate} + \text{Asset’s Beta} \times \text{Equity Risk Premium}
\]

CAPM has also been criticized for capturing only the market and systematic risk, instead of the total risk, in its computation for the risk-adjusted discount rates (Zhang, 2010). The CEM alternatively adjusts the cash flow for risk, i.e., the certainty equivalent, to be discounted at the risk-free rate instead of the expected cash flow discounted at a Risk-adjusted rate derived from the CAPM. Shortly, CEM can augment the precision of the valuation process, subject to the assignment of an apt utility function, which indeed is a talk ask (Robichek & Myers, 1966).
Certainty Equivalent of the Risky Ijārah Payoff

The design of a Sharī‘ah-compliant instrument, solicits an intricate treatment and an eye for details. Instead of raising the discount rate by adding a premium to the required rate of return (discussed in the earlier section), which may be deemed ṭabā‘ī, this study attempts to replace the risky cash flows by their ascertained cashflows called ’Certainty Equivalent’ (C.E.) (Gregory, 1978). This in-effect is the bare minimum expected cash flow, which is equally attractive to the presumably risk-averse investor as compared to the risky prospect (Becker & Sarin, 1987).

Therefore, the lease payoffs are modelled under the C.E. framework with a Von Neumann log utility function to emulate a risk-averse investor. The decreasing marginal utility with increasing wealth levels, represented by the Concave log utility function, is consistent with the implied risk-averseness of the investor. Figure 10 elucidates the risk-averse investor’s utility curve and the notion of C.E. being the wealth level corresponding to the expected utility of the risky proposition.

Assuming a three-state (and two periods) probability distribution, instead of a sophisticated continuous probability distribution, wherein the lease payoff in regular, default, and prepayment state are represented by \( L_r, L_d, \) and \( L_p \) with probability distribution.

![FIGURE 5. Log Utility Function, Expected Payoffs, Utility and Certainty Equivalent](image)

![FIGURE 6. Stylized Representation of Expected Payoff and their Probability Distribution in a Two-Period Ijārah Šukūk](image)
The expected lease payoff of the various states, described above can be mathematically expressed as follows:

**Time 1 - Expected Payoffs**

\[ L_1 = (1 - p_1 - q_1)(L' + DT) + p_1(1 - K)(L' + DT) + q_1(L' + DT + P_0 - D_{econ}) \]

**Time N (Period / Non Terminal) - Expected Payoff (Assuming \( p_n = x_n \) and \( q_n = y_n \))**

\[ L_n = (1 - p_n - q_n)n(L' + DT) + (1 - p_n - q_n)n - 1p_n(1 - K)(L' + DT) + (1 - p_n - q_n)n - 1q(n)(L' + DT + P_0 - nD_{econ}) \]

**Time 2 (Terminal) - Expected Payoffs**

\[ L_2 = (1 - p_1 - q_1)(1 - x_2 - y_2)(L' + DT + P_0 - D_{econ}) + (1 - p_1 - q_1)x_2(1 - K)(L' + DT + P_0 - D_{econ}) + (1 - p_1 - q_1)y_2(L' + DT + P_0 - D_{econ}) \]

\[ L_2 = (L' + DT + P_0 - D_{econ}[(1 - p_1 - q_1)(1 - x_2 - y_2) + (1 - p_1 - q_1)x_2(1 - K) + (1 - p_1 - q_1)y_2] \]

**Time N (Terminal) - Expected Payoff (Assuming \( p_n = x_n \) and \( q_n = y_n \))**

\[ L_n = (L' + DT + P_0 - D_{econ})[(1 - p_n - q_n)n + (1 - p_n - q_n)n - 1p_n(1 - K) + (1 - p_n - q_n)n - 1q_n] \]

Based on the already assumed investor’s utility function and the probability distribution of the three-state payoffs, the expected utility of the investor for the risky lease payoff for time period \( t = 1 \), is as follows:

\[ E[U(W_1)] = (q)U(L_p) + (P)U(L_d) + (1 - p - q)U(L_r) \]

Where, expected utility for ‘n’ period can be represented as,

\[ E[U(W_n)] = (1 - p - q)^{(n-1)}(q)U(L_p) + (1 - p - q)^{(n-1)}(p)U(L_d) + (1 - p - q)^nU(L_r) \]

Lastly, the C.E. equivalent is computed by taking invert of log function as demonstrated below:

\[ \text{Log}(CE) = E[U(W)] \]

\[ CE = e^{E[U(W)]=} \]

**Numerical Illustration**

To illustrate the concept of C.E., as the ascertained lease payoff which furnishes the same
utility to the investor as that of a risky investment - i.e., it is the minimum payoff acceptable to the investor to take on the risk exposure. To exemplify, a hypothetical case of a 4 period or two-year lease is taken, with four semi-annual rental payments and liquidation of the leased asset at maturity in the 4th period. L’ is the post-tax rental payoff (with depreciation write-offs) and only incorporates the terminal cash flow in the 4th period. With log function emulating investor utility, the expected utilities and respected certainty equivalent have been produced in the table below:

<table>
<thead>
<tr>
<th>Assumptions</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Destruction Factor K</td>
<td>K</td>
<td>1%</td>
</tr>
<tr>
<td>Rental + Dep Shield Per Period</td>
<td>L’+DT</td>
<td>4.75</td>
</tr>
<tr>
<td>Prepayment Payoff for Time 1</td>
<td>L’+ DT + Qo - Decon1</td>
<td>114.2</td>
</tr>
<tr>
<td>Prepayment Payoff for Time 2</td>
<td>L’+ DT + Qo - Decon2</td>
<td>113.65</td>
</tr>
<tr>
<td>Prepayment Payoff for Time 3</td>
<td>L’+ DT + Qo - Decon3</td>
<td>113.1</td>
</tr>
<tr>
<td>Prepayment Payoff for Time 4</td>
<td>L’+ DT + Qo - Decon4</td>
<td>112.55</td>
</tr>
<tr>
<td>EconDep per period D</td>
<td>D</td>
<td>0.50%</td>
</tr>
<tr>
<td>Regular</td>
<td>1-p1-q1</td>
<td>99.25%</td>
</tr>
<tr>
<td>Default*</td>
<td>p1 = x1</td>
<td>0.50%</td>
</tr>
<tr>
<td>Prepayment</td>
<td>q1 = y2</td>
<td>0.25%</td>
</tr>
<tr>
<td>Initial ijārah Investment P0</td>
<td>P0</td>
<td>100</td>
</tr>
<tr>
<td>ijārah Asset Value Q</td>
<td>Q</td>
<td>110</td>
</tr>
<tr>
<td>Haircut</td>
<td>Q-P0</td>
<td>10</td>
</tr>
</tbody>
</table>

![FIGURE 7. Stylized Representation of Certainty Equivalent of individual payoffs of a 4 period Ijārah Sukūk Payoffs](image)

Here, a four-period lease ijārah sukūk is illustrated in the picture above, to be disintegrated/STRIPED into four Zero Coupon Bond, i.e., three Coupon STRIPs and one coupon plus terminal value STRIP. The yields of the Zeros, thus created as represented $r_1, r_2, r_3,$ and $r_4$.  


TABLE 3
Numerical Illustration of the Calculation of Certainty Equivalents of Individual Payoffs of a 4 period Ijārah Sukūk

<table>
<thead>
<tr>
<th>Time</th>
<th>Lr</th>
<th>Ld</th>
<th>Lp</th>
<th>E(L')</th>
<th>U(Lr)</th>
<th>U(Ld)</th>
<th>U(Lp)</th>
<th>E[U(W)]</th>
<th>CE=eE[U(W)]</th>
<th>Price of the Sukūk STRIP discounted at Risk-Free Spot rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time 1</td>
<td>4.71</td>
<td>0.02</td>
<td>0.29</td>
<td>5.02</td>
<td>1.55</td>
<td>-3.75</td>
<td>-1.25</td>
<td>1.52</td>
<td>4.56</td>
<td>4.386</td>
</tr>
<tr>
<td>Time 2</td>
<td>4.68</td>
<td>0.02</td>
<td>0.28</td>
<td>4.98</td>
<td>1.54</td>
<td>-3.75</td>
<td>-1.27</td>
<td>1.51</td>
<td>4.52</td>
<td>4.172</td>
</tr>
<tr>
<td>Time 3</td>
<td>4.64</td>
<td>0.02</td>
<td>0.28</td>
<td>4.95</td>
<td>1.54</td>
<td>-3.76</td>
<td>-1.28</td>
<td>1.5</td>
<td>4.49</td>
<td>3.938</td>
</tr>
<tr>
<td>Time 4</td>
<td>109.21</td>
<td>0.55</td>
<td>0.28</td>
<td>110.03</td>
<td>4.69</td>
<td>-0.6</td>
<td>-1.29</td>
<td>4.65</td>
<td>104.78</td>
<td>87.49</td>
</tr>
<tr>
<td>Summation</td>
<td>99.986</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*S&P and Moodys published historical default probability ranges 0.5 to 0.6 for AAA-rated bonds

CONCLUSION

The goal of financial markets is to fit into the liquidity preferences of investors, be it a bank deposit contracts, credit lines or secondary markets instruments, to furnish the allocative flexibility risk and liquidity requirements. The novel Islamic Money Market product presented in this study is an endeavor towards achieving this goal. The study presents a holistic model for designing and valuing ijārah based STRIP securities in a non-Ribawi framework. The valuation model owes its uniqueness in its ability to simultaneously incorporate the endogenous microeconomic utility function of investor (using Certainty equivalent methodology), along with the integration of exogenously determined required rate of return (by bootstrapping method), as dictated by the term structure and macroeconomic dynamics in an economy. We found that the proposed ISS can serve as a short-term liquidity management instrument, facilitate price discovery and define the term structure in Islamic debt markets. For sovereign issues, ISS can enable efficient monetary policy transmission, market completeness, and public finances for the underdeveloped Muslim world. Given the absence of Islamic money market and liquidity management instruments, the ISS proposed in this study can potentially be a valuable addition to the arsenal of Islamic Financial Markets, which can augment the much needed financial deepening and development. In terms of shariah compliance, ISS instruments presented in this study are founded on a ijārah (lease) transaction, which fulfills the fundamental doctrine of Shari‘ah, and curbs agency cost of debt, and hence economically efficient (Ebrahim et al., 2015). The paper articulates the Usuli rationalization based on the overarching maqāṣid (aims) of Islamic law for the proposed ijārah sukūk STRIP instruments.

The valuation model has not incorporated the tax treatment, which can be a subject of future research, is pertinent to mention. Besides, the determination and optimization of utility functions to emulate investor behavior in Muslim countries can also be probed. As the scope of this study was limited to the ijārah securities with predetermined profit, leaving a gap for potential research to the design of mudārārah and sukūks based sukūks STRIPs.
REFERENCES


doi: https://doi.org/10.1007/s11782-010-0015-1