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PRIMARY RESEARCH

Can Real Options Reduce Moral Hazards in Profit and Loss Sharing Contracts?: A Behavioural Approach Using Game Theory and Agent Based Simulation

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Abstract. In this paper, we try reducing the moral hazard of profit misreporting in Profit and Loss Sharing Contract (PLS). In this kind of contracts , the corporate manager has a temptation to misreport profits which can lead to either project failing or to financiers receiving an unfair allocation of profits. To help in solving this problem we propose a new model that includes a real option that gives the corporate manager (agent) the right, but not the obligation, to gradually buy shares in the corporation from the financier/bank. We compare our results with the standard case of PLS without real options. We show, using a multi-agent simulation (Netlogo) that embedding real options in the PLS contract can reduce the profit misreporting case. The fact that PLS contracts are riskier compared to other forms of financing such as debt, provides an incentive for the creation of models that reduce their risk to capital providers. Given the results obtained from our real options model, the latter could prove to be of practical use to financial institutions willing to engage in PLS financing.

KAUJIE Classification: 131, O1
JEL Classification: C6, C7, C15

INTRODUCTION

PLS contracts are forms of partnership whereby two or more partners share in capital and or labour to undertake a project. In this specific mode of financing, the profit share is predetermined and is denoted in a ratio or percentage of profits. PLS contracts do not guarantee return on investments (Hesse & Jobst, 2008) making them different from fixed income securities. The losses, however, must be shared with respect to each participant percentage share in the partnership capital.

PLS are the fundamental differentiation between Islamic banks and conventional ones (Chong & Liu, 2009; Hamza & Saadaoui, 2013) . In this context, there is no interest bearing

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deal with some specific forms of partnership called *muḍārabah* or *mushārakah* (Mirakhor & Zaidi, 2007). *Mushārakah* is a partnership where the financier/bank and the entrepreneur jointly participate in the project with capital. hand, in *muḍārabah*, it is only the financier who finances the project while the entrepreneur provides labour and management. Both forms are financial instruments which are akin to equity provided by banks to entrepreneurs. As *mushārakah* and *muḍārabah* are based on the sharing of profits as well as sharing losses, they are of particular concern to banks (Mirakhor & Zaidi, 2007) and financiers. This makes these forms riskier relative to conventional debts. For example, under *mushārakah* contracts, if the project fails then the bank/financier jointly loses the invested capital along with the entrepreneurs. Even worse, the bank/financier solely loses the invested capital under *muḍārabah*. This makes *muḍārabah* financing to be perceived as bearing more risks compared to *mushārakah* (Ariffin *et al.,* 2009; Louhichi & Boujelbene, 2016; Said *et al.,* 2013). On the other hand, under conventional lending, banks are guaranteed interest payments as well as capital invested. They are further immune when they impose collateral against their funding. The second risk of *mushārakah* and *muḍārabah* contracts is that the banks return is entirely linked to the rate of return of the projects being financed. If the projects are low then the bank/financier suffers lower profitability. On the other hand, under the conventional system, the bank/ financier is concerned mainly about being paid a guaranteed interest payment even if the rate of profitability of projects is low.

The third risk is effort shirking. Under *mushārakah* , and even more under *muḍārabah*, the fact that losses are shared may lead to entrepreneurs' effort shirking as there is no obligation to redeem any compensation to the financier. However, under conventional financing, interest payments are not tied to profits and are therefore mandatory payments.

The fourth risk, which is the substance of this paper, is profit misreporting. Under *mushārakah* and *muḍārabah*, the entrepreneur oversees managing the project and has complete knowledge of the profitability of the projects. The entrepreneur can act opportunistically by hiding, misreporting, part of the project profitability, and only share a falsely declared profit with the bank/financier. This in turn puts more burden on the bank/financier to put costly mechanisms to minimize the misreporting risk.

The fifth risk we can relate to PLS contracts such as *mushārakah* and *muḍārabah* is credit risk (Warninda *et al.,* 2019) work on *muḍārabah* claims that the highest level of credit risk is achieved when *mushārakah* contracts constitute 37-39% of the bank's financing.

But despite their credit riskiness PLS contracts in both forms have got defenders. Defenders of PLS contracts argue that their profit and risk-sharing makes them preferred to conventional debt contracts (Ebrahim & Safadi, 1995). This is supported by Dar and Presley (2000) who argue that there is no justification for the claims that PLS are inefficient. Opposers to PLS contracts, however, argue that despite their overwhelming advantages, they suffer heavily from the presence of asymmetric information such as profit misreporting. They also argue that, contrary to debt contracts, there is less protection from losses to the financiers as they cannot claim guarantees against losses. This is true if managers enjoy the upsides of investment and are protected against its failures (Cornelli & Yosha, 2003). In such a delegated investment framework, Harris et al. ()1982) claims agents are tempted to misreport profits to have the rest of the undeclared profits for personal use. In such a framework, managers have a temptation for empire building by diverting free cash flows inappropriate investments (Bernardo *et al.,* 2001; Harris & Raviv, 1996). This issue of profit misreporting makes PLS contracts less of a preferred mode of financing (Zaher & Kabir Hassan, 2001) to financiers in comparison to other modes such as debt contracts (Aggarwal & Yousef, 2000; Ahmed, 2002; Khan, 1985; Khan, 1986).

Some solutions were proposed to reduce the problem of misreporting in PLS contracts. One way is randomized monitoring which is argued to be the most effective way of preventing the underreporting problem Khan (1985). In line with monitoring, higher due diligence is proposed in PLS in comparison with conventional PLS (Al-Suwailem, 2006). Our model is different from those two arguments in the following way. Monitoring itself is costly. In our model, the corporate manager is induced to properly report profits as he gradually owns the corporation for which he is paying the real option premium.

Another method is employing the corporate manager under a low job protection scheme. Under this method, a manager offers to be employed under low job terms to signal his managerial confidence. This approach is criticised for its unfairness as projects' failure can be due to factors beyond the agents' scope of control (Elfakir & Tkiouat, 2015b).

There have been many works that tried to reduce asymmetric information in PLS contracts. For example, the agent's financial participation in the project's capital (Karim, 2002) may reduce moral hazard. This work is consistent with the work of Nabi (2013) who suggests agents to put in a minimum capital as well as be awarded a minimum profit share. This is also in line with other research (Elfakir & Tkiouat 2015a; Elfakir & Tkiouat, 2015b) which rationally claims that under the PLS contract, moral hazard is reduced as both agents face the same destiny of losing their capital. This is also consistent with Inness (1990) who claims that contracts with a sharing arrangement are not feasible when they are unilaterally financed.

Those approaches are like ours, in terms of capital sharing, but do not treat the problem of profit misreporting. We propose the introduction of 'Diminishing PLS contracts' whereby the share of the financier is purchased gradually by the corporate manager (Usmani, 2002). In our opinion, this represents an efficient mechanism of reducing profit misreporting risk. In fact, as the financier gets his ownership of the corporation reduced, the risks of the projects are also gradually transferred to the corporate manager. In addition to this, we propose that to buy the share of the financier, the corporate manager pays a real option premium. The right to buy is conditioned by the fact that the firm must reach a minimum threshold value. This encourages the corporate manager to truly report profit if she wants to gain total ownership in a short time.

Given the above literature, profit misreporting seems to be a challenging aspect in PLS contracts corporate governance compared to standard debt. This paper tries to reduce this risk by combing PLS contracts with real options.

The agents' behaviour (in our case the financier/bank and the corporate manager), besides the dynamic relationship between the two, are complex phenomena. For this reason, we make recourse to two techniques: 1) game theory and 2) agent-based simulation (explained in the next section). The contribution of this work manifest itself in three ways: First, PLS contracts are more complex than debt contracts; so standard debt contracting methods do not apply in this framework. Second, we believe that no previous work has been done to introduce real options in PLS analysis. Third, this work is unique as the work is tested using a personally programmed agent-based model. The latter allows for a user-friendly interface whereby the financier can input project key data and calculate easily the related output (option premium, firm value, exit point, monetary incentives etc.)

The rest of this paper will proceed in the following way:

Part 2 explains agent-based simulation and why we use it in this study. Part 3 explains the model. Part 4 identifies the mythology adopted. Part 5 introduces the results and provides a discussion; and finally, we conclude with summary and further venues of extension.

Agent Based Modelling (ABM)

Rational of using ABM

ABM is a computational methodology and simulation that provides more flexible ways to understand complex problems from the perspective of its behavioural entities (Bonabeau, 2002; Wilensky & Rand, 2015). Rand and Rust (2011) note that a complex environment can be modelled by describing simple characteristics of agents' behaviour. ABM allows the organized behaviour of a system (emergent phenomena) to be observed without being explicitly encoded in the simulation (Xiang *et al.,* 2005). The ABM can replicate real systems (Yahyaoui & Tkiouat, 2018) or explore phenomena that may not even exist in the real world. It enables researchers to discover what will happen when they assume a few basic rules (Wilensky & Rand, 2015). Some researchers have gone so far as to consider ABM as a new way of doing science using computer-based experiments (Axelrod, 1997; Macal & North, 2006; Wilensky and Rand, 2015). Naciri and Tkiouat (2015) confirm that ABM can be used to solve many practical problems where other modelling tools show their limitation. Equation Based Modelling (EBM) is a top-down approach that usually studies the global behaviour of the modelled system, but not the reasons locally leading to that behaviour. However, the ABM is a bottom-up approach; it can model the interactions between individual agents and explain the phenomena resulting from these interactions (Naciri & Tkiouat, 2015). Therefore, the ABMs provide more detailed results than EBMs (Wilensky & Rand, 2015). Indeed, ABM naturally describes a system (Bonabeau, 2002) as its concept is much closer to natural thinking (Wilensky & Rand, 2015). ABM also can embed features difficult to incorporate in an analytical model. In an agent-based model, individuals can have bounded rationality, be heterogeneous, adaptive and located within geographical space (Rand & Rust, 2011).

The use of ABM can be different from one model to another; it can be used for giving a simplified description of real or artificial phenomena, explaining the mechanisms and phenomenon that control a system, doing experiments for understanding an engineered system, or for making predictions (Wilensky & Rand, 2015).

The rationales cited above are useful for our work in two ways: 1) the relationship between financier/bank and the manager is complex from a behavioural point. In other words, the manager will always have the temptation to misreport profits while the financier wants to maximize his wealth but minimize the chances of misreporting. 2) Since the relationship is a dynamic one, it is hard to track and predict performance and misreporting behaviour over time. Therefore, a simulation tool is needed for this purpose. In our case, we are going to use Netlogo[1], a famous and widely used tool for agent-based simulation.

Fundamental Steps in ABM

An agent-based model refers to a model that is composed of agents. Wilensky and Rand (2015) define agents as autonomous decision-making entities with properties and behavioural rules in a computer simulation. They, agents, include consumers, producers, entrepreneurs, institutions (banks), governments, markets, etc. Based on their behavioural rules, agents can interact with each other and/or with their environment (Garcia, 2005). For instance, if we consider an entrepreneur as an agent, it might have properties such as "project", "wealth" and "productivity" as well as the decision-making process; for example, an entrepreneur agent might have the rule to select a new project. Macal and North (2006) emphasize that typically, an agent is characterized by his/her independent decision-making ability, ranging from simple to complex rules. According to the literature, agent-based modelling nictitates the respect of certain fundamental steps:

1. Decide if ABM is appropriate: ABM can be a useful choice when the problem under investigation emphasizes a set of autonomous and heterogeneous entities (agents) evolving over time and when both their micro-level behaviour and the result of their interactions (macro-level patterns) are of interest (Garcia ,2005; Rand & Rust, 2011; Wilensky & Rand, 2015).

2. Design the model: the modeler needs to identify the agents of the system, their properties, their behaviours, and the environment where they live.

3. Implement the model in computational software: Many ABM toolkits exist such as SWARM, Mason, REPAS and NetLogo.

4. Validate and verify the model: To evaluate an agent-based model and increase confidence in its simulation, two activities are generally conducted; Verification and Validation (V&V). Verification refers to the process of determining whether the simulation code corresponds to the conceptual model (Zou *et al.,* 2014) i.e. the program does what it is supposed to do (David, 2006). Validation determines if the conceptual framework and the simulation output accurately represent the real word (Zou *et al.,* 2014).

5. Using the model as a decision-making tool: Once an agent-based model is verified and validated, it can be used for running a series of computational experiments based on different combinations of inputs. The generated results can be analyzed using standard statistical methods.

[1]NetLogo is a multi-agent programmable modelling environment. https://ccl.northwestern.edu/netlogo/

THE MODEL

Previously we tried to reduce asymmetric information between a bank and a risk-neutral corporate manager who requires funding a project costing F. He is endowed with personal wealth f and requires additional resources F-f to complete his project's funding requirement (Elfakir & Tkiouat, 2016).

The project is at the risk of being profit (Π_t) misreported with a degree θ_t ε[01]. The rate of return of the project is given as r.

The manager has an opportunity cost $c\%$ given as a percentage of his capital contribution. On hand the financier/bank has an opportunity cost of ρ. We can then note the expected output as:

$$E\left(\frac{\Pi_t}{\theta_t}\right) = \theta_t \Pi_t = \theta_{t.}\left(1 + r_t\right).F \tag{1}$$

The financier share of the project is given as α, while his loss share is β_t. The specific nature of the PLS contract dictates that the % loss β_t of each partner should not exceed their % contribution in the capital.

We extend this model by having the corporate manager gradually buying the part of the bank in the project conditioned on the buying of a real option premium.

We look at scenarios where the real option is used and the case where it is not used. To facilitate the calculations, we refer to $Y = 1$ and $Y = 0$ as the case where the real option is used and not used respectively. Taking a conventional number of shares as 100, the value of the real option is determined using the Black and Scholes formula as:

$$R_{t-1} = max\left\{O; \left(S.N(d1) + E.N(d2).e^{r\int}\right).n_t\right\} \tag{2}$$

Where E and S are the exercise price and the spot price and are given simply as:

$$E = \frac{F}{100} \tag{3}$$

$$S = \frac{r_t.F}{WAAC_t.100} \tag{4}$$

$WAAC_t$ is the weighted average cost of capital at each round t and is given as:

$$WAAC_t = \beta_t.\rho + (1 - \beta_t).c \tag{5}$$

N is the cumulative standard normal , $N(d1)$ and $N(d2)$ are probability factors and distribution function with θ as the volatility of returns such that:

$d2 = \frac{log\frac{S}{E} - \left(r - 1/2\sigma^2\right).t}{\sigma.\sqrt{t}}$ and $d2 + \sqrt{t}$

n_t is the number of shares to be purchased by the corporate manager from the bank at the end of each round t and it is given as:

$$n_t = min\left\{\alpha_t.100; \frac{r_t.(1-\alpha_t).F}{E}\right\} \tag{6}$$

The Symmetric Case

In this case, both agents/manager and bank, cooperate, i.e. the bank charges a lower sharing ratio and the manager truly report the profits. Consequently, the expected bank's profit is:

$$E\left(\Pi_t^b\right) = [\alpha_t(1+r_t) - (1+\rho)\beta_t]F + R_{t-1}.Y_{t-1} \tag{7}$$

On the other hand the corporate manager profit is:

$$E\left(\Pi_t^{Corp}\right) = [(1-\alpha_t)(1+r_t) - (1+c)(1-\beta_t)]F + R_{t-1}.Y_{t-1} \tag{8}$$

The Asymmetric Case

Under this case, both participants are unaware of their counterpart's hidden potential strategies: i.e cooperate or defect. To solve this problem we use a repeated game approach and simulate its results using a multiagent simulation (Netlogo). The strategies available for each participant are:

- Cooperate: The financier/bank would cooperate by taking a small profit sharing ratio α_{lt} while the agent manager truly reports profits.
- Defect: The bank requests a high profit ratio α_{ht} which includes a misreporting risk premium R_p, α_{ht}=$\alpha_{ht}.\left(1+R_{pt}\right)$. The manager on the other hand falsely reports profits.

METHODOLOGY

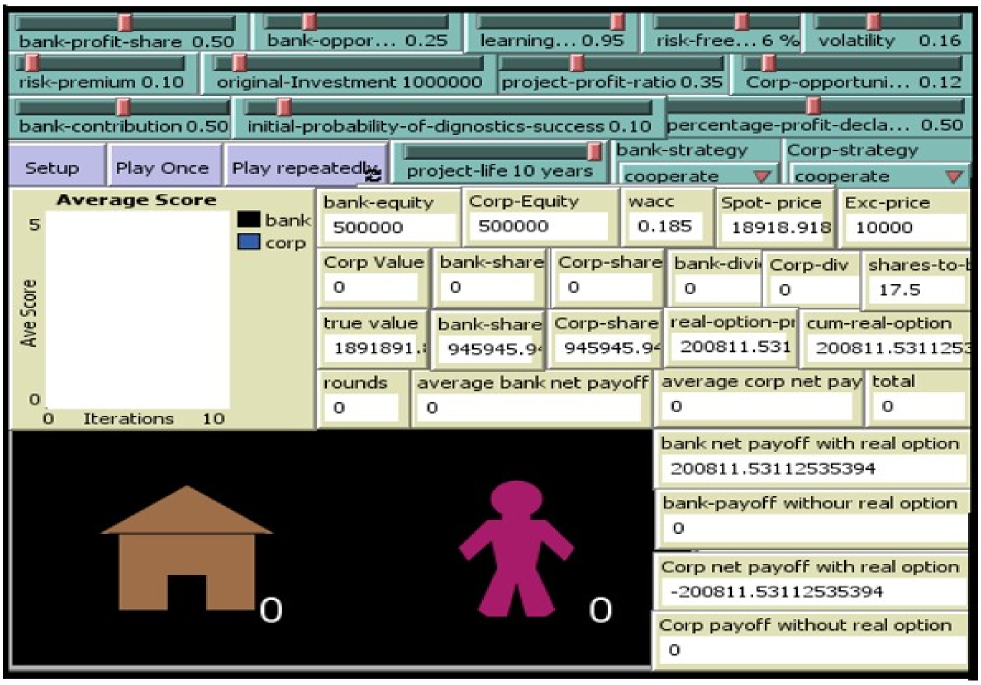

FIGURE 1. The Netlogo interface under some initial parameters

We simulate our work in an agent-based simulation platform, (see Fig. 1). The figure also shows the starting parameters used to initiate the simulation.

Under each case we provide the periodic, average, and cumulative payoffs of each participant.

Both Agents Cooperate

Under this scenario, the periodic, cumulative and average bank's payoffs are given respectively as:

$$E\left(\Pi_t^b\right) = [\alpha_{lt}(1-r_t) - (1+\rho)\,\beta_t]F + [R_{t-1}.Y_{t-1}] \tag{9}$$

$$E_{cum}\left(\Pi_t^b\right) = \sum\nolimits_{t=0}^{N}[\alpha_{lt}(1-r_t) - (1+\rho)\,\beta_t]F + \sum_{i=1}^{n}[R_{t-1}.Y_{t-1}] \tag{10}$$

$$E_{ave}\left(\Pi_t^b\right) = \frac{\sum_{t=0}^{N}[\alpha_{lt}(1-r_t) - (1+\rho)\,\beta_t]F + \sum_{i=1}^{n}[R_{t-1}.Y_{t-1}]}{n} \tag{11}$$

Similarly, the periodic, cumulative and average manager's payoffs are given respectively as:

$$E\left(\Pi_t^{Corp}\right) = [(1-\alpha_{lt})(1+r_t) - (1+c)(1-\beta_t)]F - R_{t-1}.Y_{t-1} \tag{12}$$

$$E_{cum}\left(\Pi_t^{Corp}\right) = \sum\nolimits_{t=0}^{N}[(1-\alpha_{lt})(1+r_t) - (1+c)(1-\beta_t)]F - \sum\nolimits_{t=0}^{N} R_{t-1}.Y_{t-1} \tag{13}$$

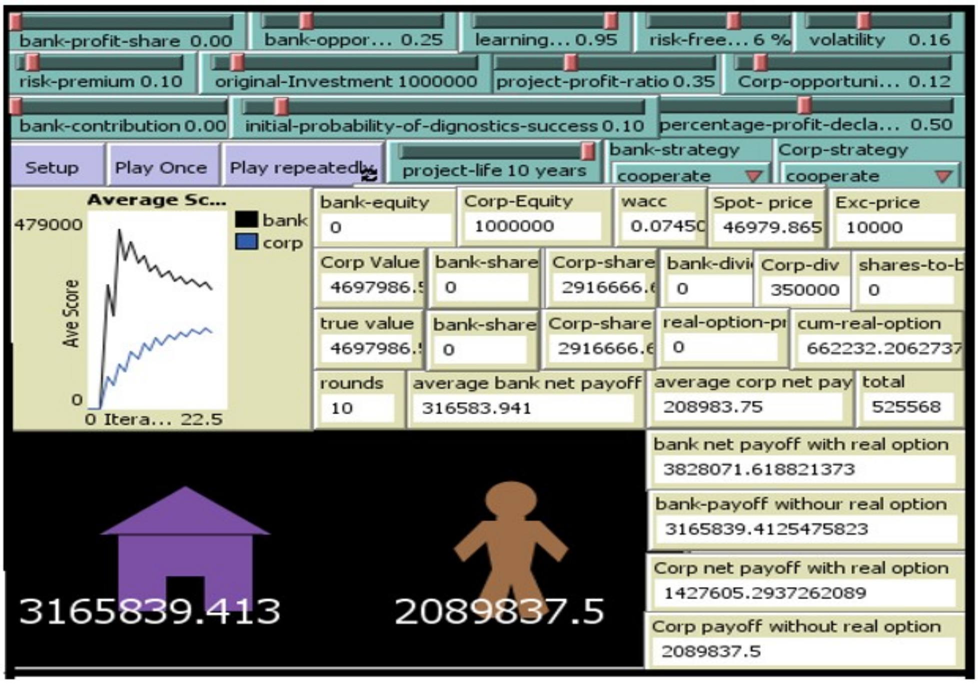

FIGURE 2. The Netlogo interface under some initial parameters under both the bank and the corporate manager cooperating

$$E_{ave}\left(\Pi_t^{Corp}\right) = \frac{\sum_{t=0}^{N}[(1-\alpha_{lt})(1+r_t)-(1+c)(1-\beta_t)]F-\sum_{t=0}^{N}R_{t-1}.Y_{t-1}}{N} \tag{14}$$

For convenience, we provide the Interface of the Netlogo simulation for this result:

The Corporate Manager Cooperates and the Financier/Bank Defects

If the bank cooperates while the corporate manager defects, then the periodic, cumulative and average payoff at each round to the bank are given respectively as:

$$E\left(\Pi_t^b\right) = [\alpha_t(1+r_t)\theta_t-(1+\rho)\beta_t]F+R_{t-1}.Y_{t-1} \tag{15}$$

$$E_{cum}\left(\Pi_t^{\int}\right) = \sum_{t=0}^{N}[\alpha_l(1+r_t)\theta_t-(1+\rho)\beta_t]F+\sum_{t=0}^{N}R_{t-1}.Y_{t-1} \tag{16}$$

$$E_{ave}\left(\Pi_t^b\right) = \frac{\sum_{t=0}^{N}[\alpha_l(1+r_t)\theta_t-(1+\rho)\beta_t]F+\sum_{t=0}^{N}R_{t-1}.Y_{t-1}}{N} \tag{17}$$

Similarily, the periodic, cumulative and average manager's payoffs are given respectively as:

$$E\left(\Pi_t^{Corp}\right) = [\alpha_l(1+r_t\theta_t)(1+r_t)-(1+C)(1-\beta_t)]F+\sum_{t=0}^{N}R_{t-1}.Y_{t-1} \tag{18}$$

$$E_{cum}\left(\Pi_t^{Corp}\right) = \sum_{t=0}^{N}[\alpha_l(1+r_t\theta_t)(1+r_t)-(1+C)(1-\beta_t)]F-\sum_{t=0}^{N}R_{t-1}.Y_{t-1} \tag{19}$$

$$E_{ave}\left(\Pi_t^{Corp}\right) = \frac{\sum_{t=0}^{N}[\alpha_l(1+r_t\theta_t)(1+r_t)-(1+C)(1-\beta_t)]F-\sum_{t=0}^{N}R_{t-1}.Y_{t-1}}{N} \tag{20}$$

The Bank Defects and the Corporate Manager Cooperates

If the bank defects while the corporate manager cooperates, then the periodic, cumulative and average payoff at each round to the bank are given respectively as:

$$E\left(\Pi_t^b\right) = [\alpha_{ht}(1+r_t)-(1+\rho)\beta_t]F+R_{t-1}.Y_{t-1} \tag{21}$$

$$E_{cum}\left(\Pi_t^b\right) = \sum_{t=0}^{N}[\alpha_{ht}(1+r_t)-(1+\rho)\beta_t]F+\sum_{t=0}^{N}R_{t-1}.Y_{t-1} \tag{22}$$

$$E_{ave}\left(\Pi_t^b\right) = \frac{\sum_{t=0}^{N}[\alpha_{ht}(1+r_t)-(1+\rho)\beta_t]F+\sum_{t=0}^{N}R_{t-1}.Y_{t-1}}{N} \tag{23}$$

Similarily, the periodic, cumulative and average manager's payoffs are given respectively as:

$$E_{cum}\left(\Pi_t^{Corp}\right) = [(1-\alpha_{ht})(1+r_t) - (1+c)(1-\beta_t)]F - R_{t-1}.Y_{t-1} \tag{24}$$

$$E_{cum}\left(\Pi_t^{Corp}\right) = \sum_{t=0}^{N}[(1-\alpha_{ht})(1+r_t) - (1+c)(1-\beta_t)]F - \sum_{t=0}^{N}[R_{t-1}.Y_{t-1}] \tag{25}$$

$$E_{ave}\left(\Pi_t^{Corp}\right) = \frac{\sum_{t=0}^{N}[(1-\alpha_{ht})(1+r_t) - (1+c)(1-\beta_t)]F - \sum_{t=0}^{N}[R_{t-1}.Y_{t-1}]}{N} \tag{26}$$

Both Participants Defect

In this case, the periodic, cumulative and average payoff at each round to the bank are given respectively as:

$$E\left(\Pi_t^{Corp}\right) = [\alpha_{ht}(1+r_t)\theta_t - (1+\rho)\beta_t]F \tag{27}$$

$$E_{cum}\left(\Pi_t^{Corp}\right) = \sum_{t=0}^{N}[\alpha_{ht}(1+r_t)\theta_t - (1+\rho)\beta_t]F + \sum_{t=0}^{N}[R_{t-1}.Y_{t-1}] \tag{28}$$

$$E_{ave}\left(\Pi_t^{Corp}\right) = \frac{\sum_{t=0}^{N}[\alpha_{ht}(1+r_t)\theta_t - (1+\rho)\beta_t]F + \sum_{t=0}^{N}[R_{t-1}.Y_{t-1}]}{N} \tag{29}$$

Similarily, the periodic, cumulative and average manager's payoffs are given respectively as:

$$E\left(\Pi_t^{Corp}\right) = [(1-\alpha_{ht})\theta_t(1+r_t) - (1+c)(1-\beta_t)]F - [R_{t-1}.Y_{t-1}] \tag{30}$$

$$E_{cum}\left(\Pi_t^{Corp}\right) = \sum_{t=0}^{N}[(1-\alpha_{ht})\theta_t(1+r_t) - (1+c)(1-\beta_t)]F - \sum_{t=0}^{N}[R_{t-1}.Y_{t-1}] \tag{31}$$

$$E_{ave}\left(\Pi_t^{Corp}\right) = \frac{\sum_{t=0}^{N}[(1-\alpha_{ht})\theta_t(1+r_t) - (1+c)(1-\beta_t)]F - \sum_{t=0}^{N}[R_{t-1}.Y_{t-1}]}{N} \tag{32}$$

Using Netlogo we provide the calculation of each participant payoff. The calculations take into consideration the opportunity cost of each agent in undertaking the project.

RESULTS AND DISCUSSION

We use a 10 period timscale and simulate our model using the decision parameters. In each case, we identify the payoffs to the participants in the cases of diminishing PLS with real option (RO^+) and without real option (RO^-). Also, we identify the Nash Equilibrium of each case. In case the Nash Equilibrium is found, we identify the social value as the total of the participant's payoffs.

To encourage the corporate manager to accept the real option, we introduce an incentive λ

such that the corporate manager is indifferent between accepting the real option or not. We have then:

$$\lambda = Max\left\{O; E_{cum}\left(\Pi_t^{corp}/RO^+\right) - E_{cum}\left(\Pi_t^{corp}/RO^-\right)\right\} \tag{33}$$

We start by initial parameters as our starting point of the simulatio:$\theta_t = 30\%; \alpha = 50\%; R_p = 20\%; r = 35\%; p = 25\%; c = 12\%; \beta_t = 60\%; F = 100000$. At each simulation, we change a parameter at a time while maintaining the rest of the initial parameters as constants: The results of our simulation are shown in the appendix figures. The first simulation of the model under the initial values is given below. The other simulations of the model under the changes in the values of the initial parameters are given in the appendix.

r	θ	L	α		c	ρ		F	Rf	б	β	Rp
35%	50%	95%	50%		12%	25%		1000000	6%	16%	50%	10%

		WITH REAL OPTION				WITHOUT REAL OPTION			
		Corporation				Corporation			
		Cooperate		Defect		Cooperate		Defect	
	Cooperate	3828071	1427605	2244726	1115732	3165839	2089837	1140649	2219809
Bank	Defect	3973933	1345979	2326532	1022519	3276131	2043871	1161119	2187932
		*NE =	DC	**SV =	5319912	*NE =	DD	**SV=	3349051

Incentive to accept real-option = 841953

	Corporation			
	Accept		Not Accept	
Real option	3131980	2187932	1161119	2187932
	**SV=	5319912	3349051	

FIGURE 3. The game under initial parameters of the simulation

The Fig. 4 shows a summary of the simulation results:

	Parametre		Nash Equilibrium		Social Value		λ	Social value with incentive
	Initial Value	New Value	RO+	RO-	RO+	RO-		
r	35%	60%	DC*	DD**	9721471	6413441	1681791	9721471
θ	50%	30%	DC	DD	5319912	2497999	964456	5319912
c	12%	7%	DC	DD	6460792	3771764	1157622	6460792
ρ	25%	15%	DC	DD	5307548	3003626	1137279	5307548
F	1000000	2000000	DC	DD	10639825	6698103	1683907	10639825
Rf	6%	2%	DC	DD	5275754	3349051	819875	5275754
б	16%	30%	DC	DD	5298176	3349051	831086	5298176

*Bank Defecting, Corporation cooperating

**Bank Defecting, Corporation Defecting

FIGURE 4. Summary table of the simulation results

It is clearly apparent that under each simulation a Nash Equilibrium emerges. Yet this equilibrium depends on whether we use a real option or not.

If we do not use a real option, Nash equilibrium emerges as DD in which case the bank defects by charging a high profit sharing ratio and the corporate manager defects by misreporting. This is a bad equilibrium state for two reasons: 1) If the bank takes a higher profit sharing ratio, this can deter safe entrepreneurs and only attract riskier ones. This is similar to a conventional banking sector where credit rationing happens when a bank takes a higher interest payment which deters safe entrepreneurs and attracts riskier ones (Bester, 1985a, 1985b).

If we use a real option, a Nash equilibrium emerges as DC in which case the bank defects by charging a high sharing ratio, and the corporate manager cooperates by not misreporting. Under this case, a higher social value emerges compared to the case where we do not use real options.

While this case, with real options, still has the case of the bank defecting by taking a higher share, it is better than the case without real options where the entrepreneur's best strategy is to misreport.

Since the case of real option results in the corporate manager cooperating and results in a higher social value, we would like the corporate manager to accept the real option contract. To do so we introduce a monetary incentive mechanism. Our agent-based simulation proves that a high social value is maintained while inducing the corporate manager to cooperate.

Conclusion

In this work, we attempted to reduce profit misreporting in PLS contracts by embedding real options. We have made recourse to game theory and agent-based simulation. We compare the performance of PLS combined with real options and the performance of PLS without real options. The simulation result shows that under real options a Nash Equilibrium exists when the bank charges a higher sharing ratio (Defect) and the corporate manager cooperates by truly reporting profits. Under no real option, however, a Nash equilibrium exists under the bank charging a high ratio (Defect) and the corporate manager defecting by misreporting. The simulation evidence shows a higher social value when real options are embedded in the PLS contracts. An incentive is proposed to foster this high social value. We found evidence that accepting the real option results in a higher social value than if the corporate manager refuses the real option. Due to the agency problem of profit misreporting, this model can be a good corporate governance tool for financiers in general, and banks in particular, willing to engage in PLS contracts as an equity investment. The results of this model have policy implications. Indeed, we have shown how real options can reduce the risk of profit misreporting. Since this is one of the main moral hazards risk in PLS contracts, its reduction may lead many financial institutions to increase their offer of PLS contracts.

Another policy implication is that conventional systems may be more motivated to adopt PLS contracts. While we do not expect conventional banks to eradicate interest-bearing funding but their adoption of PLS contracts, with our real options model, may reduce interest-bearing trading. This should, therefore, induce conventional systems to have an active engagement in productive activities rather than being passive under lending's transactions.

Another policy implication is that our model may increase the PLS offer of Islamic banks relative to their offer of other 'debt-like' products such as Ijara (leasing) or Murabaha.

There are further venues to extend this study. First, this model can be extended to include different types of financiers (VC's and Angels, Conventional banks), along with the ones engaged in PLS contracts. The inclusion of different financiers will add more realism to the complexity of the model and could be a good reflection of the real market case. In the second extension, we can compare our PLS model with another conventional setting which charges interests. The purpose would assess where misreporting is likely to happen.

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1 Appendix: Simulation results under changes of the initial values of the model parameters

r	θ	L	α		c	ρ		F	Rf	б	β	Rp
60%	50%	95%	50%		12%	25%		1000000	6%	16%	50%	10%

WITH REAL OPTION

		Corporation: Cooperate		Corporation: Defect	
Bank	Cooperate	6558756	3026081	4057923	2361426
	Defect	6823857	2897614	4195994	2217946
		*NE =	DC	**SV =	9721471

WITHOUT REAL OPTION

		Corporation: Cooperate		Corporation: Defect	
Bank	Cooperate	5120837	4464000	1800250	4619100
	Defect	5315671	4405800	1834036	4579405
		*NE =	DD	**SV=	6413441

Incentive to accept real-option = 1681791

	Corporation: Accept		Corporation: Not Accept	
Real option	5142066	4579405	1834036	4579405
**SV=	9721471		6413441	

FIGURE 5. The game under an increase in the project-profit-ratio from r=35% to 60%

r	θ	L	α		c	ρ		F	Rf	б	β	Rp
35%	30%	95%	50%		12%	25%		1000000	6%	16%	50%	10%

WITH REAL OPTION

		Corporation: Cooperate		Corporation: Defect	
Bank	Cooperate	3828071	1427605	2124956	641255
	Defect	3973933	1345979	1969620	528396
		*NE =	DC	**SV =	5319912

WITHOUT REAL OPTION

		Corporation: Cooperate		Corporation: Defect	
Bank	Cooperate	3165839	2089837	433467	2332744
	Defect	3276131	2043781	187564	2310435
		*NE =	DD	**SV=	2497999

Incentive to accept real-option = 964456

	Corporation: Accept		Corporation: Not Accept	
Real option	3009477	2310435	187564	2310435
**SV=	5319912		2497999	

FIGURE 6. Figure 5: The game under a decrease in the percentage profit declared from θ = 50% to 30%

r	θ	L	α	c	ρ	F	Rf	б	β	Rp
35%	50%	95%	50%	7%	25%	1000000	6%	16%	50%	10%

WITH REAL OPTION

		Corporation			
		Cooperate		Defect	
Bank	Cooperate	4818849	1551658	2805872	981685
	Defect	5007769	1453023	2916763	855001
		*NE =	DC	**SV =	6460792

WITHOUT REAL OPTION

		Corporation			
		Cooperate		Defect	
Bank	Cooperate	3826357	2544150	1140649	2646909
	Defect	3965335	2495438	1161119	2610645
		*NE =	DD	**SV=	3771764

Incentive to accept real-option = 1157622

	Corporation			
	Accept		Not Accept	
Real option	3850147	2610645	1161119	2610645
**SV=	6460792		3771764	

FIGURE 7. The game under a decrease in the corporate manager's opportunity cost from c=12% to 7%

r	θ	L	α	c	ρ	F	Rf	б	β	Rp
35%	50%	95%	50%	12%	15%	1000000	6%	16%	50%	10%

WITH REAL OPTION

		Corporation			
		Cooperate		Defect	
Bank	Cooperate	4057651	1148203	2355869	650390
	Defect	4256895	1050653	2475640	527986
		*NE =	DC	**SV =	5307548

WITHOUT REAL OPTION

		Corporation			
		Cooperate		Defect	
Bank	Cooperate	3116017	2089837	786450	2219809
	Defect	3263467	2043781	815694	2187932
		*NE =	DD	**SV=	3003626

Incentive to accept real-option = 1137279

	Corporation			
	Accept		Not Accept	
Real option	3119616	2187932	815694	2187932
**SV=	5307548		3003626	

FIGURE 8. The game under a decrease in the bank's opportunity cost from ρ=25% to 15%

r	θ	L	α	c	ρ	F	Rf	б	β	Rp
35%	50%	95%	50%	12%	25%	2000000	6%	16%	50%	10%

WITH REAL OPTION

		Corporation			
		Cooperate		Defect	
Bank	Cooperate	7656143	2855210	4489452	2231465
	Defect	7947867	2691958	4653064	2045039
		*NE =	DC	**SV =	10639825

WITHOUT REAL OPTION

		Corporation			
		Cooperate		Defect	
Bank	Cooperate	6331678	4179675	2281298	4439619
	Defect	6552262	4087562	2322238	4375865
		*NE =	DD	**SV=	6698103

Incentive to accept real-option = 1683907

	Corporation			
	Accept		Not Accept	
Real option	6263960	4375865	2322238	4375865
**SV=	10639825		6698103	

FIGURE 9. The game under an increase in the investment from F=1000000 to 2000000

r	θ	L	α	c	ρ	F	Rf	б	β	Rp
35%	50%	95%	50%	12%	25%	1000000	2%	16%	50%	10%

		WITH REAL OPTION				WITHOUT REAL OPTION			
		Corporation				Corporation			
		Cooperate		Defect		Cooperate		Defect	
Bank	Cooperate	3765212	1448558	2209412	1151046	3123933	2089837	1140649	2219809
	Defect	3907697	1368057	2289111	1059940	3231973	2043781	1161119	2187932
		*NE =	DC	**SV =	5275754	*NE =	DD	**SV=	3349051

Incentive to accept real-option = 819875

	Corporation			
	Accept		Not Accept	
Real option	3087822	2187932	1161119	2187932
**SV=	5275754		3349051	

FIGURE 10. The game under a decrease in the risk free rate from R_f =6% to 2%

r	θ	L	α	c	ρ	F	Rf	б	β	Rp
35%	50%	95%	50%	12%	25%	1000000	6%	30%	50%	10%

		WITH REAL OPTION				WITHOUT REAL OPTION			
		Corporation				Corporation			
		Cooperate		Defect		Cooperate		Defect	
Bank	Cooperate	3793354	1439177	2238717	1121741	3142694	2089837	1140649	2219809
	Defect	3941330	1356846	2321428	1027623	3254395	2043781	1161119	2187932
		*NE =	DC	**SV =	5298176	*NE =	DD	**SV=	3349051

Incentive to accept real-option = 831086

	Corporation			
	Accept		Not Accept	
Real option	3110244	2187932	1161119	2187932
**SV=	5298176		3349051	

FIGURE 11. The game under a increase in the volatility of return from θ=16% to 30%

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